



Mathematics teacher support material

Example interim objectives



Objectives for years 1, 3 and 5 of the Middle Years Programme

Year 5 objectives

The mathematics objectives for year 5 of the Middle Years Programme (MYP) are already in place and can be found in this guide. This set of **prescribed** objectives forms the basis for the **assessment criteria**, also published in the guide, which must be used for final assessment of students' work during year 5.

Example interim objectives

Example interim objectives for years 1 and 3 of the MYP appear in the tables that follow. They have been developed in order to:

- promote articulation between the MYP and the Primary Years Programme (PYP)
- support individual schools in developing a coherent curriculum across the five years of the programme (or however many years a school is authorized to offer)
- emphasize the need to introduce students to the required knowledge, understanding, skills and attitudes from the first year of the programme
- provide examples of possible learning experiences that will allow students to work towards meeting the final objectives for year 5
- support schools that are authorized to offer the first three years of the MYP in designing appropriate assessment tasks for the end of the third year.

Unlike the objectives for year 5, the interim objectives for years 1 and 3 are not prescribed, although the IB recommends that all schools use them. Schools may choose to adopt the objectives contained in this document or develop their own.

If choosing to develop their own interim objectives, schools must start with the prescribed objectives for year 5 and modify each one by taking into account the age, prior knowledge and stage of development of students in an earlier year of the programme. Each year 5 objective will then correspond directly to a modified objective in a preceding year of the programme. **No objectives should be omitted** from a previous year as it is vital to ensure a coherent progression of learning across all five years of the programme.

MYP units of work

Examples of possible learning experiences, each aligned to an objective, appear in the tables that follow. Each learning experience is intended to form part of a larger unit of work designed to address a central question or theme, known as the **MYP unit question**. More information about MYP units of work can be found in the section on "Planning for teaching and learning" in *MYP: From principles into practice* (August 2008).

Within each unit of work, the **context for learning**, **significant concept(s)** and **assessment tasks** are defined in relation to the MYP unit question. The areas of interaction provide the context for learning while the significant concepts refer to the underlying concepts that define the principal goal of the unit. Assessment tasks are designed to address the levels of students' engagement with the MYP unit question and the aligned objectives.

Context for learning

Every MYP unit of work has an approaches to learning (ATL) component: a shared and agreed set of skills that all teachers develop with their students throughout the entire programme. The context that frames a particular unit of work is generally derived from one of the other four areas of interaction, although ATL might be the specific context on some occasions. Some of the examples of learning experiences listed in the tables that follow have an obvious connection to one of the areas of interaction, for example, investigating the relationship between the volume of air in a classroom (and other enclosed spaces) and the health requirements of each student. Other connections may become clear only after a more considered approach but teachers should be able to establish these connections for their own students within each MYP unit of work.

Several examples of learning experiences listed also strongly suggest the possibility of planning an interdisciplinary unit in collaboration with other subject teachers, for example, representing Newton's laws of motion as algebraic equations, tables and graphs using data that has been generated and data that has been collected experimentally.

Assessment tasks

One of the first stages in planning a unit of work is to design **summative assessment tasks**, linked to the MYP unit question, which provide varied opportunities for students to demonstrate their knowledge, understanding, skills and attitudes. It is also important to include ongoing **formative assessment tasks** within a unit of work as these provide valuable insights into the extent of student learning as the unit of work progresses.

It is important to realize that the formats of both summative and formative assessment tasks need not be reduced to examinations, tests, quizzes and written questions set as homework. These formats are valid in certain cases but do not always take into account different learning styles and may not provide students with sufficient creative scope to demonstrate all they have learned. There are many different ways in which evidence of student learning can be found. For example, students could carry out assessment tasks that involve:

- making a presentation using visual aids (for example, flipcharts, electronic slides)
- solving a cross-number puzzle where the clues are provided in the form of calculations to be made and/or problems to be solved
- playing a game that requires a particular set of skills or knowledge and understanding of certain concepts
- making a three-dimensional model (for example, scale models of the earth, moon and sun)
- telling a story (for example, stories where numbers have been deliberately scaled up or down by multiples of ten to provide comic entertainment, thereby demonstrating the need for accuracy with regard to place value)
- keeping a personal journal that documents their development of mathematical understanding
- making a poster or wall chart
- writing a short song or poem that incorporates important mathematical principles (for example, a rap chant incorporating the principles of Pythagoras' theorem)
- creating a mnemonic as an aid to memory (for example, the rules for the sine, cosine and tangent properties of a right-angled triangle expressed in one word, SOHCAHTOA, or as a phrase or saying, "Some owls have")
- keeping a scrapbook containing extracts from the media that illustrate a particular property
- maintaining a folder of their own work
- developing an information booklet/leaflet that describes a concept and/or mathematical process in detail
- writing a summary sheet as a revision guide for a particular mathematical topic
- creating pictures, diagrams or cartoons to illustrate a particular concept or process
- carrying out an investigation
- collecting data and storing it in appropriate formats (for example, tables, spreadsheets)
- creating a personal data booklet.

Tables of objectives

A Knowledge and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop problem-solving skills. Through knowledge and understanding students develop mathematical reasoning to make deductions and solve problems.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to demonstrate basic knowledge and understanding of the following branches of mathematics:	At the end of the third year, students should be able to demonstrate some knowledge and understanding of the following five branches of mathematics:	At the end of the course, students should be able to demonstrate knowledge and understanding of the following five branches of mathematics:
<ul style="list-style-type: none"> number algebra geometry and trigonometry statistics and probability discrete mathematics by being able to:	<ul style="list-style-type: none"> number algebra geometry and trigonometry statistics and probability discrete mathematics by being able to:	<ul style="list-style-type: none"> number algebra geometry and trigonometry statistics and probability discrete mathematics by being able to:
<ul style="list-style-type: none"> know and demonstrate understanding of some of the basic concepts of number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics 	<ul style="list-style-type: none"> know and demonstrate understanding of some of the concepts of number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics 	<ul style="list-style-type: none"> know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
<ul style="list-style-type: none"> use basic concept-specific strategies to solve simple problems in both familiar and unfamiliar situations including those in real-life contexts 	<ul style="list-style-type: none"> use appropriate mathematical concepts and skills to solve simple problems in both familiar and unfamiliar situations including those in real-life contexts 	<ul style="list-style-type: none"> use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real-life contexts
<ul style="list-style-type: none"> apply basic rules correctly to solve simple problems including those in real-life contexts. 	<ul style="list-style-type: none"> select and apply basic rules correctly to solve problems including those in real-life contexts. 	<ul style="list-style-type: none"> select and apply general rules correctly to solve problems including those in real-life contexts.
Examples of possible learning experiences		
Number Students could: <ul style="list-style-type: none"> find data in newspapers and classify it as discrete or continuous classify numbers as natural, odd, even, square and/or triangular use the Sieve of Eratosthenes to find prime numbers less than 100 	Number Students could: <ul style="list-style-type: none"> make a geological time line on the wall of the classroom investigate the relationship between the volume of air in a classroom (and other enclosed spaces) and the health requirements of each student 	Number Students could: <ul style="list-style-type: none"> compare the number of kilometres per litre of fuel per passenger used by planes, trains, buses and/or cars explore the history and significance of irrational numbers and identify some of the symbols used for particular irrational numbers, for example, π (pi), e (Euler's number), ϕ (the golden ratio).

<ul style="list-style-type: none"> count very large sets of objects (coins, cars passing, people in a large space, numbers) to emphasize the importance of organization and grouping make a time line on the wall of the classroom emphasizing dates of important mathematicians and mathematical discoveries. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> use a pan balance to simulate the addition or subtraction of like quantities from both sides of an equation by keeping the pans balanced use the balance model to create equations for classmates to solve create pictures on squared paper and provide a list of coordinates for classmates to construct the picture by joining the points in order create a booklet containing information on algebra topics (explanations, examples and exercises) for use by subsequent students. <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> investigate the sum of the interior angles of triangles and quadrilaterals by drawing and cutting out different shapes on paper, tearing off the angles and fitting them together to form straight lines and/or circles investigate the tangent ratio by comparing students' heights with the lengths of their shadows investigate reflections by using a mirror to reflect faces along different centre lines. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> find the ranges and means of leaf lengths on two trees of the same species but located in different environments (sun and shade). 	<ul style="list-style-type: none"> investigate rounding numbers to a specified number of significant figures by considering the accuracy of measurements in real life, such as the length of a 100 m athletics track, the extent of possible errors and the impact these may have. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> generate a series of ordered pairs by substituting values in a linear equation and have classmates identify patterns and/or work out the formula find a rule for the number of 1 cm squares needed to put a 1 cm-wide frame around a square picture whose side is n cm. <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> draw or construct a model of an appropriate building (the school or their own house) by piecing together rectangular or triangular prisms only estimate the volumes of irregular solid objects as the sum of more simple approximated shapes and verify their results by immersing each object in water and measuring the displaced volume investigate the underlying patterns and constructions evident in particular designs and/or artworks, for example, traditional Moroccan designs, and then create their own tessellated designs use Pythagoras' theorem and the trigonometric ratios as tools for measuring large-scale objects and/or distances in open spaces. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> design statistical surveys to investigate health and social education issues, with guidance from the teacher construct a tree diagram for a three-day weather forecast where the probability of rain on any day is estimated from past data compare the lengths of words and/or sentences in texts aimed at different readerships. 	<p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> show that the ratios of successive terms of a Fibonacci sequence (u_n/u_{n+1}) converge to the same value regardless of the term chosen as u_1 find the best angle for throwing a basketball so that it will go in the basket from the free throw line, by modelling its trajectory graphically using their knowledge of quadratic equations investigate exponential growth in a biological population. <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> use the unit circle as a physical tool to calculate the values of different trigonometric ratios (in order to appreciate the circular nature and symmetry of each function, and the significance of the asymptotes in the tangent function) use the transformations of translation, reflection, rotation, enlargement and shear to describe the actions of a particular cartoon character carry out research into the history of angle measurement and the introduction of trigonometry. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> collect information relating to used cars for sale (mileage, age, make, engine size, cost now, cost when new) and explore the relationships between different pairs of variables select several countries and look for key statistics (population growth, average income and life expectancy) on the Internet in order to answer questions such as, "Do people in richer countries appear to live longer?", "Is there any relation between the size of a population and its average income?"
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<p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> draw two large intersecting circles on the floor and identify two categories of student, for example, {girls} and {students wearing something blue}, and move into one of the four defined regions according to their characteristics play mathematical games (for example, “bingo”, by calculating the answers to simple problems read out by the teacher) to review and reinforce previous learning. 	<p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> create a minimum network for broadband cables to connect five major cities in their country create a Koch snowflake by starting with a large equilateral triangle, dividing each side into three equal parts, removing the middle part and replacing it with two sides of a triangle equal in length to the part that was removed, and so on. 	<p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> solve a logical puzzle, such as the following: “You have three boxes of fruit, one with apples, one with oranges and one mixed; each box is labelled but the labels are not on the correct boxes of fruit—how can you know what each box contains simply by taking one piece of fruit from one box?” create their own logic puzzle by using websites such as http://www.edhelper.com/logic_puzzles.htm) play the chaos game (refer to http://home.inreach.com/kfarrell/fractals.html).
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B Investigating patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Mathematical inquiry encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate and/or analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, when investigating problems, in both theoretical and real-life contexts, students should be able to:	At the end of the third year, when investigating problems, in both theoretical and real-life contexts, students should be able to:	At the end of the course, when investigating problems, in both theoretical and real-life contexts, students should be able to:
<ul style="list-style-type: none"> • apply basic inquiry and mathematical problem-solving techniques, with guidance from the teacher, by identifying variables, posing relevant questions, organizing data and using an appropriate model 	<ul style="list-style-type: none"> • select and apply basic inquiry and mathematical problem-solving techniques to problems by asking searching questions 	<ul style="list-style-type: none"> • select and apply appropriate inquiry and mathematical problem-solving techniques
<ul style="list-style-type: none"> • recognize simple patterns similar to previously seen examples 	<ul style="list-style-type: none"> • recognize simple patterns in different situations 	<ul style="list-style-type: none"> • recognize patterns
<ul style="list-style-type: none"> • describe simple patterns in words and/or diagrams 	<ul style="list-style-type: none"> • describe simple patterns as relationships or general rules 	<ul style="list-style-type: none"> • describe patterns as relationships or general rules
<ul style="list-style-type: none"> • arrive at a result or set of results and make predictions based on extending the pattern(s) 	<ul style="list-style-type: none"> • arrive at a single result or set of results and make predictions consistent with findings 	<ul style="list-style-type: none"> • draw conclusions consistent with findings
<ul style="list-style-type: none"> • describe simple mathematical relationships. 	<ul style="list-style-type: none"> • explain simple mathematical relationships and general rules using logical arguments. 	<ul style="list-style-type: none"> • justify or prove mathematical relationships and general rules.
Examples of possible learning experiences		
<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> • predict the next three numbers in a sequence • find large numbers that can be reduced to prime factors by their classmates • investigate how many different designs can be made by shading squares in a 3 x 3 square. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> • investigate the graphs of $y = mx + c$ using appropriate software • measure and plot the extended length of a spring against the weight of an object hung on the end 	<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> • determine ways of finding the sum of the terms in an arithmetic sequence, describing their methods in general terms • investigate the meaning of negative exponents using a calculator • investigate the patterns present in the Fibonacci sequence. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> • investigate the graphs of $y = (x - a)(x - b)$ and their solutions for different values of y using a graphic display calculator. 	<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> • be given the diameter of a model of the earth (a globe) as 100 cm, and investigate the diameters of corresponding models of the planets, given their true diameters expressed in standard form ($a \times 10^n$), and discuss the status of Pluto as a planet. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> • analyse and compare male and female record times in a particular sport (running, swimming) from 1900 to the present day, and predict times for the year 2050 by investigating lines or curves of best fit for the data collected.

<ul style="list-style-type: none"> investigate problems such as the following: "How long does it take to make n pieces of toast on a one-sided grill that can take two pieces at a time?" <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> investigate the sum of the interior angles of different n-sided polygons ($n > 2$) use different instruments to measure a range of objects and discuss the ease and accuracy of each technique. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> investigate the probabilities for the different outcomes when tossing two coins or rolling two dice. <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> use Venn diagrams to analyse different aspects of after-school activities. 	<p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> write a logical explanation for the sum of the exterior angles of a polygon equalling 360° investigate how to measure tall structures, telegraph poles, buildings use geometry to predict the angle of the noontime sun at the two solstices. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> choose, with logical explanations, which measure of central tendency would be most appropriate for typical family size, height of students, amount of pocket money. <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> investigate shortest paths in local networks (bus routes, metro lines) investigate the concept of map colouring by conducting research into the four-colour problem and attempting to illustrate this rule by colouring a map of their country showing the different regions/states. 	<p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> carry out a survey of the school grounds for the purpose of creating a detailed and accurately scaled map/plan investigate how a sphere can be projected onto a plane. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> design a questionnaire to elicit data reflecting attitudes to issues relevant to them; circulate this questionnaire to students in their own school and one other school, possibly in another country; then collect and compare the data create a fund-raising game of chance that is profitable and will attract players. <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> investigate whether, when you start with an odd number a, square it to give b, subtract 1 to give c, divide by 2 to give d, and add 1 to give e, the equation $a^2 + d^2 = e^2$ will always be satisfied.
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C Communication in mathematics

Mathematics provides a powerful and universal language. Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings—both orally and in writing.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to communicate mathematical ideas, reasoning and findings by being able to:	At the end of the third year, students should be able to communicate mathematical ideas, reasoning and findings by being able to:	At the end of the course, students should be able to communicate mathematical ideas, reasoning and findings by being able to:
<ul style="list-style-type: none"> use appropriate mathematical language (notation, symbols, terminology) in both oral and written communications, with guidance from the teacher 	<ul style="list-style-type: none"> use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations in familiar situations 	<ul style="list-style-type: none"> use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations
<ul style="list-style-type: none"> use different forms of mathematical representation (simple formulae, diagrams, tables, charts, graphs and models), with guidance from the teacher 	<ul style="list-style-type: none"> use different forms of mathematical representation (simple formulae, diagrams, tables, charts, graphs and models) 	<ul style="list-style-type: none"> use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
<ul style="list-style-type: none"> state, in writing and/or verbally, the steps followed in solving simple problems. 	<ul style="list-style-type: none"> communicate a mathematical line of reasoning in solving simple problems using different forms of representation. 	<ul style="list-style-type: none"> communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating complex problems.
Examples of possible learning experiences		
<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> modify quantities and measurements in well-known events or stories to create absurdities and then act out or model the event or story, for example, speed limits being set at 50 m per hour (instead of 50 km per hour); playing on a football pitch where it is assumed that $1 \text{ m}^2 = 100 \text{ cm}^2$ investigate discounts in advertisements in order to determine the best deals. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> explain the steps involved in solving a linear equation explain the significance of m and c when the line represented by $y = mx + c$ is graphed. 	<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> collect media clippings of the inappropriate use of mathematical symbols and terminology, and/or inappropriate representations of data (for example, misleading labelling on graphs) and describe how these could lead to incorrect interpretations. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> use motion recording equipment to create distance/time graphs create feasible distance/time graphs and physically model the graphs created by others by moving around the classroom. 	<p>General</p> <p>Students could:</p> <ul style="list-style-type: none"> take it in turns to summarize the important elements of selected lessons by putting the information into a class file or posting it on a website in the form of a blog; the document could then be used as a revision tool by the class. <p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> be given a series of real-life calculations to carry out using a calculator (for example, “A third of the people in a city with a population of 500,000 live in poverty—how many people live in poverty?”) and be asked to justify the number of significant figures given in their answers.

<p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> • make a scale drawing of a bicycle • investigate the properties of similar two-dimensional shapes (triangles, squares and circles). <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> • measure the lengths of their ears and represent the data in tables and graphs, and find measures of central tendency • investigate the probability of various events taking place based on available data (for example, the probability of rain on a particular day of the year). <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> • use Venn diagrams to classify quadrilaterals that have equal sides, parallel lines, equal angles • draw diagrams showing three different routes they could take to travel from home to a specific destination and determine the best one by considering either times or distances. 	<p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> • write out directions, using bearings and distances, to describe the cycle routes in their local area • carry out a survey of their school grounds/campus in order to create an accurate plan. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> • collect data from a weather station and represent it graphically as a tool for investigating trends • discuss the advantages and disadvantages of obtaining data by sampling large populations, by referring to sampling techniques used by the media (for example, opinion polls). <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> • investigate the similarities and differences between the skills needed for two different occupations using Venn diagrams (for example, music teacher and rock star) • create flow charts to describe some simple mathematical processes (for example, finding the greatest common divisor of two numbers). 	<p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> • be given an open-ended problem with fixed and variable costs, such as budgeting for an event, where the solution depends on the parameters, expected outcomes and accuracy of estimates • investigate Newton's laws of motion in the form of algebraic equations, tables and graphs using generated data and data that has been collected experimentally • investigate and describe the trajectory of small objects falling from different forms of transport (bicycles, lorries/trains, hot-air balloons). <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> • design an orienteering course on the sports field where the beginning and end points coincide, and use practical trials and/or the sine and cosine rules to demonstrate that these points are coincident. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> • investigate different methods of sampling large populations (random sampling, stratified sampling, systematic sampling, cluster sampling, convenience sampling) and create a poster explaining each one. <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> • conduct a poll among themselves to determine their preferences for different types of music; collate the results in the form of a Venn diagram; then describe the information displayed.
<p><i>Students are encouraged to choose and use information and communication technology (ICT) tools as appropriate and, where available, to enhance communication of their mathematical ideas. ICT tools can include calculators (simple, scientific and/or graphic display), screenshots, graphing, spreadsheets, databases, and drawing and word-processing software.</i></p>		

D Reflection in mathematics

MYP mathematics encourages students to reflect upon their findings and problem-solving processes. Students are encouraged to share their thinking with teachers and peers and to examine different problem-solving strategies. Critical reflection in mathematics helps students gain insight into their strengths and weaknesses as learners and to appreciate the value of errors as powerful motivators to enhance learning and understanding.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to:	At the end of the third year, students should be able to:	At the end of the course, students should be able to:
<ul style="list-style-type: none"> consider the reasonableness of their results in the context of the problem 	<ul style="list-style-type: none"> consider the reasonableness of their results in the context of the problem and attempt to explain whether they make sense 	<ul style="list-style-type: none"> explain whether their results make sense in the context of the problem
<ul style="list-style-type: none"> consider the importance of their findings, with guidance from the teacher 	<ul style="list-style-type: none"> consider the importance of their findings 	<ul style="list-style-type: none"> explain the importance of their findings
<ul style="list-style-type: none"> distinguish between measurement and counting, and demonstrate an appreciation of the difference between degrees of error in measuring and mistakes in counting, measuring and calculating 	<ul style="list-style-type: none"> consider the degree of accuracy of their results where appropriate and estimate errors in simple measurements 	<ul style="list-style-type: none"> justify the degree of accuracy of the results where appropriate
<ul style="list-style-type: none"> consider alternatives to the method when appropriate, with guidance from the teacher. 	<ul style="list-style-type: none"> consider alternatives to the method when appropriate. 	<ul style="list-style-type: none"> suggest improvements to the method when necessary.
Examples of possible learning experiences		
<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> estimate answers before carrying out calculations discuss the precision of different measuring instruments (rulers, callipers, protractors, theodolites). <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> substitute answers in original problems to check results. <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> find the least and greatest amount of paint necessary to paint their bedrooms. 	<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> investigate how well the “golden ratio” applies to famous buildings or paintings. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> determine the domain and/or range of functions involving physical processes, for example, the elastic limit of a rubber band as a linear function within a limited domain. <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> design an economical shape for a 500 ml bottle of soda use Pick’s theorem to estimate the area of regions on a map 	<p>Number</p> <p>Students could:</p> <ul style="list-style-type: none"> discuss the appropriateness of degrees of accuracy for different data/information found in the media. <p>Algebra</p> <p>Students could:</p> <ul style="list-style-type: none"> discuss when substitution, graphing or elimination are the most appropriate strategies for solving different sets of simultaneous equations investigate whether a table-tennis ball rolled off a desk will follow a parabolic path. <p>Geometry and trigonometry</p> <p>Students could:</p> <ul style="list-style-type: none"> make a sundial and investigate the position of the shadow at different times of the year compared to the pre-calculated positions.

<p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> draw a bar chart showing the hours of television watched each day by classmates collect examples from the media of data displayed in different ways (bar graphs, pie charts, pictograms) and comment on their effectiveness. <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> invent a problem similar to the “Bridges of Königsberg”. 	<ul style="list-style-type: none"> scale up the measurements of a popular doll or toy model to determine whether their measurements lead to absurdities when their proportions are scaled to a realistic level. <p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> analyse the data collected to investigate an environmental problem. <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> create and model a traffic problem involving one-way and two-way streets. 	<p>Statistics and probability</p> <p>Students could:</p> <ul style="list-style-type: none"> investigate, by collecting appropriate data, whether the annual harvest of a particular fish species justifies the creation of conservation laws. <p>Discrete mathematics</p> <p>Students could:</p> <ul style="list-style-type: none"> devise emergency exit paths to support the swift and safe evacuation of students and staff in their school.
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